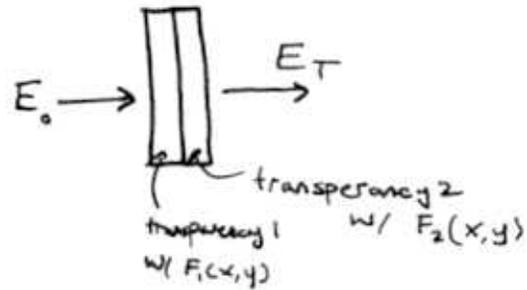


Physics 110B
Homework #11

#1 (Pedrotti 25-2)

(a) consider the electric field of a light wave incident upon these two transparencies placed in series. After transverseing the first transparency the electric field transmitted is:



$$E_T(x,y) = F_1(x,y) E_0(x,y)$$

transmission function (e.g. a polarizer matrix, etc.)

Now, this field goes through the next transparency:

$$E_T(x,y) = F_2(x,y) E_T(x,y) = F_2(x,y) F_1(x,y) E_0(x,y)$$

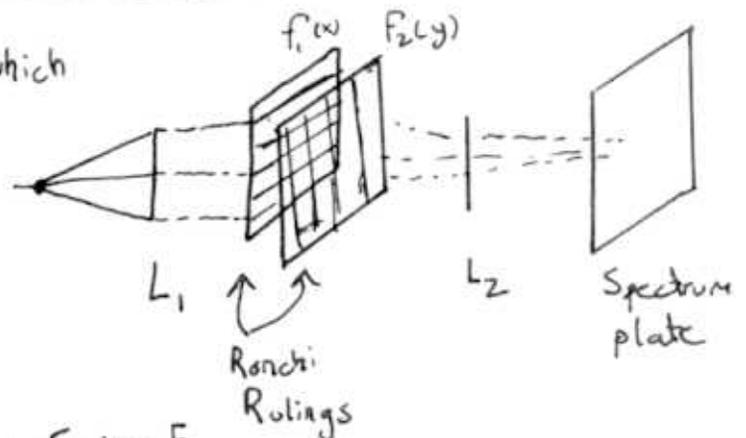
$\Rightarrow F_{Total}(x,y) = \frac{E_T}{E_0} = F_2(x,y) \cdot F_1(x,y)$

So, we take the product of the two transmission func.

(b) Here we have two Ronchi rulings, which may be represented as:

(page 526) $f_1(x) = \frac{1}{2} + c \sum_{m=1}^{\infty} \frac{\cos mx}{m}$

$$f_2(y) = \frac{1}{2} + c \sum_{n=1}^{\infty} \frac{\cos ny}{n}$$



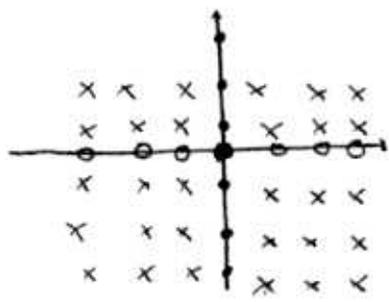
Using the above result, the total transmission Function F will be:

$$F(M,N) = \left(\frac{1}{2} + c \sum_{m=1}^{\infty} \frac{\cos mx}{m} \right) \left(\frac{1}{2} + c \sum_{n=1}^{\infty} \frac{\cos ny}{n} \right) = \text{Fourier transform of total transmission function}$$

$$= \frac{1}{4} + \frac{1}{\pi} \sum_{m=1}^{\infty} \frac{\cos mx}{m} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\cos ny}{n} + \frac{4}{\pi^2} \sum_{m,n=1}^{\infty} \frac{\cos mx \cos ny}{nm}$$

\uparrow DC \uparrow Ruling along y alone \uparrow Ruling along x alone \uparrow combinations of the two

Pattern on spectrum plate



x ~ "combinations"
 o ~ x-alone
 • ~ y-alone

#2 (Pedrotti 25-5)

$$h(x) = f(x) \otimes g(x) \Rightarrow h(x) = \int_{-\infty}^{\infty} f(y) g(x-y) dy$$

$$f(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(k) e^{-iky} dk$$

$$g(x-y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(l) e^{-il(x-y)} dl$$

$$\Rightarrow h(x) = \frac{1}{(2\pi)^2} \int dy dl dk f(k) g(l) e^{-ikx} e^{-i(k-l)y}$$

$$\int e^{i(k-l)y} dy = 2\pi \delta(k-l)$$

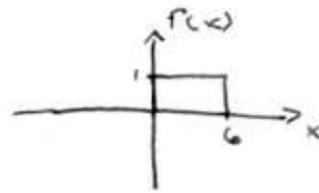
$$= \frac{1}{2\pi} \int dk f(k) g(k) e^{-ikx}$$

$$h(x) = \frac{1}{2\pi} \int h(k) e^{-ikx} dk$$

$$\Rightarrow \begin{aligned} h(k) &= f(k) g(k) \\ \text{or } \mathcal{F}(h(x)) &= \mathcal{F}(f(x)) \mathcal{F}(g(x)) \end{aligned}$$

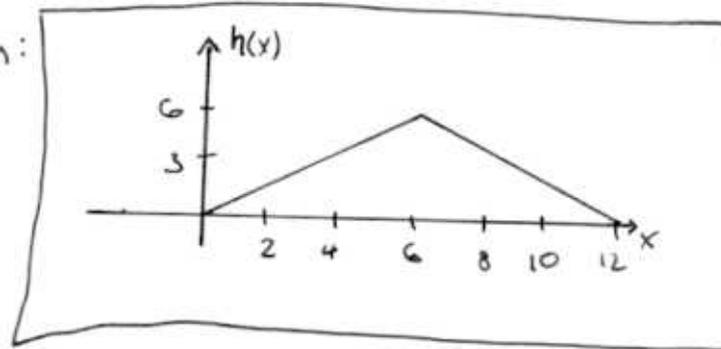
#3 (Pedrotti 25-6)

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$



$$h(x) = f(x) \otimes f(x) = \int_{-\infty}^{\infty} f(y) f(x-y) dy = \int_0^6 f(x-y) dy$$

Which, when we graph:



#4 (Pedrotti 25-7)

$$f(t) = A \sin(\omega t + \alpha)$$

Using eq(25-30) in one-dimension

$$\Phi_{11}(\tau) = \int_0^{\tau} (A \sin(\omega t + \tau) + \alpha) (A \sin(\omega t + \alpha)) dt$$

$$= A^2 \int_0^{\tau} \sin(\omega t + \alpha) (\sin(\omega t + \alpha) \cos \omega \tau + \sin \omega \tau \cos(\omega t + \alpha)) dt$$

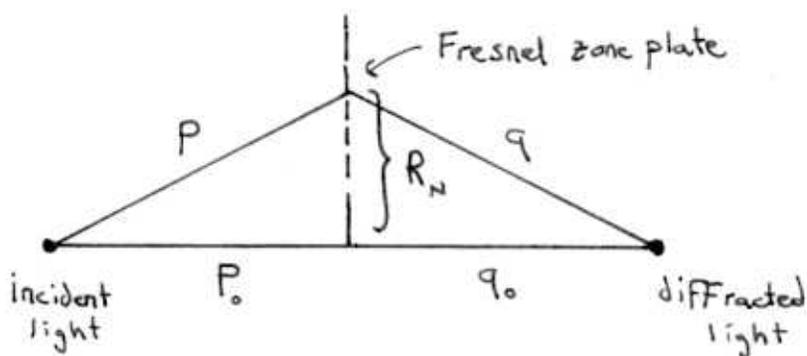
$$= A^2 \cos \omega \tau \int_{\alpha}^{2\pi + \alpha} \frac{1}{\omega} \sin^2 y dy = \frac{A^2}{\omega} \cos \omega \tau \left[\frac{y}{2} - \frac{\sin 2y}{4} \right]_{\alpha}^{2\pi + \alpha}$$

$$= \frac{A^2}{\omega} \cos \omega \tau \left(\pi + \frac{\alpha}{2} - \frac{\sin(4\pi + 2\alpha)}{4} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4} \right)$$

$$\boxed{\Phi_{11}(\tau) = \frac{\pi A^2}{\omega} \cos \omega \tau}$$

#5 (Pedrotti 18-5)

Using Figure (18-8)



For Fresnel zone, by definition the difference between the shortest path $p_0 + q_0$ and the path to the n^{th} Fresnel zone is equal to $-N\lambda/2$:

$$\textcircled{1} (P_0 + Q_0) - (P + Q) = -\frac{N\lambda}{2}$$

By the Pythagorean theorem:

$$P^2 = P_0^2 + R_N^2$$

$$Q^2 = Q_0^2 + R_N^2$$

$$P = (P_0^2 + R_N^2)^{1/2}$$

$$Q = (Q_0^2 + R_N^2)^{1/2}$$

$$= P_0 \left(1 + \frac{R_N^2}{P_0^2}\right)^{1/2}$$

$$= Q_0 \left(1 + \frac{R_N^2}{Q_0^2}\right)^{1/2}$$

$$\approx P_0 + \frac{R_N^2}{2P_0} \quad \begin{matrix} \frac{R_N}{P_0} \ll 1 \\ \text{and } \frac{R_N}{Q_0} \ll 1 \end{matrix}$$

$$\approx Q_0 + \frac{R_N^2}{2Q_0}$$

Plugging these results into eq ①

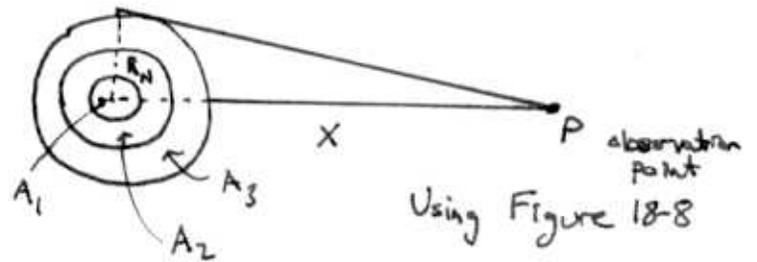
$$\Rightarrow \frac{R_N^2}{2P_0} + \frac{R_N^2}{2Q_0} = \frac{N\lambda}{2}$$

$$\Rightarrow R_N = \sqrt{\frac{N\lambda P_0 Q_0}{P_0 + Q_0}} = \sqrt{NL\lambda}$$

#6 (Pedrotti 18-11)

The area of the N^{th} Fresnel half-period zone is,

$$\text{Area}_N = A_N = \pi R_N^2 - \pi R_{N-1}^2$$



using the result for plane waves eq (18-20):

$$R_N = \sqrt{N\lambda x} \Rightarrow A_N = \pi N\lambda x - \pi(N-1)\lambda x$$

$$\boxed{A_N = \pi\lambda x}$$

#7 (Pedrotti 18-17)

Using the Cornu spiral of Figure 18-12:

- The first max. occurs at $v = -1.2$
- The second max. occurs a full cycle 2π later at $v = -2.35$

From Table 18-1 (Fresnel Integrals)

$$v = -2.35 \Rightarrow \begin{aligned} C(v) &= -.5908 & H' \\ S(v) &= -.5864 \end{aligned}$$

$$v = -\infty \Rightarrow \begin{aligned} C(-\infty) &= -.5 & E' \\ S(-\infty) &= -.5 \end{aligned}$$

The magnitude of the phasor $H'E'$ is

$$\frac{E_p}{E_0} = \left((.5908 + .5)^2 + (.5864 + .5)^2 \right)^{1/2} = 1.54$$

$$\Rightarrow \boxed{I_p = (1.54)^2 E_0^2 = 2.37 I_0 = 1.19 I_u \text{ irradiance second max.}}$$

The next min. occurs half a cycle π later at $v = -2.75$

From Table 18-1

$$v = -2.75 \Rightarrow C(-2.75) = -.3908 \quad J'$$

$$S(-2.75) = -.5015$$

Thus, the mag. of phasor $J'E'$ is,

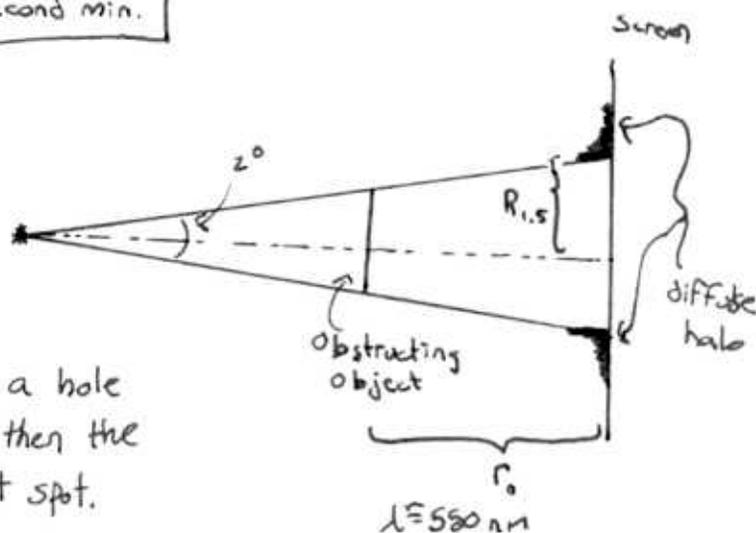
$$E_p/E_o = \left((.3908 + .5)^2 + (.5015 + .5)^2 \right)^{1/2} = 1.34$$

$$\Rightarrow I_p = 1.797 I_o = .89 I_u \quad \text{irradiance second min.}$$

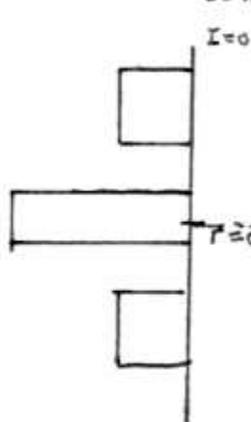
8 (Pedratti 18-21)

How many Fresnel zones should the particle cover in order to produce this halo?

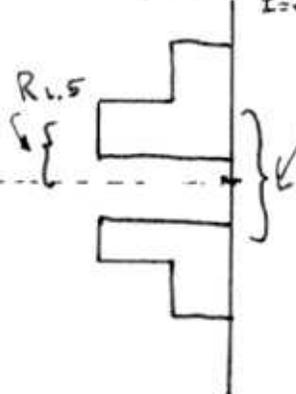
- If we have an obstructing object with a hole which is one Fresnel zone wide, then the pattern on the screen would be a bright spot.
- The complementary aperture, however, would just give a dark spot on the screen.
- However, covering slightly more than one Fresnel zone would give the correct complementary pattern of a halo.



A hole covering three Fresnel zones



Disk covering three Fresnel zones (By Babinet's principle)



This is approx. diffuse halo pattern at 1.5 Fresnel zone

So, Radius Particle $\approx R_{1.5}$

$R_{1.5}$ ~ radius between first and second Fresnel zone

$$R = R_{1.5} \approx \sqrt{1.5 \lambda r_0} \quad (\text{eq 18-20})$$

And, $\frac{R_{1.5}}{r_0} = \frac{z_0}{2} = \frac{\pi}{180}$ radians

$$\Rightarrow R_{1.5} \approx \frac{1.5 \lambda \cdot 180}{2\pi} \approx 23.6 \text{ nm}$$